Stock Price Clustering and Discreteness: The "Compass Rose" and Predictability

Constantinos E. Vorlow [a],*

[a]*Durham Business School, University of Durham, Mill Hill Lane, Durham, DH13LB, UK.*

Abstract

In this letter we investigate the information provided by the "compass rose" (Crack, T.F. and Ledoit, O. (1996), Journal of Finance, 51(2), pg. 751-762) patterns revealed in phase portraits of daily stock returns. It has been initially suggested that the compass rose is just a manifestation of price clustering and discreteness and the tick size, factors that can affect the unbiasedness of an array of statistical tests based on stock returns. We show that this may not entirely be the case.

Key words: Price Clustering and Discreteness, Microstructure, Compass rose, Nonlinear and Complex Dynamics, Surrogate Data Analysis.
(JEL G10; G12; G22; Z00).
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1 Introduction

Price clustering and discreteness is considered an important chapter of the "market microstructure" literature with serious implications for tests of market efficiency, risk evaluation techniques and optimal design of securities procedures. The "*compass rose*", introduced dy Crack and Ledoit (1996), is simply the manifestation of price clustering and discreteness in two or three dimensional phase portraits. More precisely, a pattern is usually revealed in scatter diagrams of daily percentage returns against their lagged values: rays emanating from the center of the portrait (the origin of the Cartesian axis system), generating a compass rose like formation of clusters of points.

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* Corresponding author
  Email address: Costas@vorlow.org (Constantinos E. Vorlow).
  URL: http://www.vorlow.org (Constantinos E. Vorlow).

Crack and Ledoit (1996) identify three conditions necessary for the appearance of the compass rose pattern:

(1) The daily price changes of the stock should be small relative to the price level;
(2) daily price changes should also be effected in discrete jumps of a small number of ticks and
(3) the price of the stock should vary over a relatively wide range.

If any of the above points is violated, compass rose patters will be very weak or fail to realize altogether.

Several papers have recently appeared on the compass rose theme. Some mainly confirm the compass rose as a result of the tick size, price clustering and discreteness and investigate or relax the above conditions under which it appears (see Szpiro, 1998, Wang et al., 2000, Chen, 1997, Lee et al., 1999, Gleason et al., 2000, Wang and Wang, 2002 and McKenzie and Frino, 2003) . Others (see Fang, 2002, Kramer and Runde, 1997, Koppl and Nardone, 2001 and Amilon, 2003) also concentrate on how the above factors affect the validity of statistical tests based on stock return sequences that reveal this "nanostructure" (Szpiro, 1998). The compass rose should have appeared in several other works (such as Enright, 1992, Chen, 1993, Papaioannou and Karytinos, 1995, Brealy and Meyers, 1996, Franses, 1998, Franses and Dijk, 2000 and Andreou et al., 2000). However this was not achieved due to an unfortunate choice of graphical representation style or resolution and of the length (history) of the sequences examined.

Crack and Ledoit (1996) suggested originally that the compass rose may not offer any help in predicting returns series. This is due to an apparent absence of any strong temporal continuity pattern in the phase portraits (see Fig. 1a). In this letter we show that from a small manipulation of the information that appears in the compass roses, we can gain additional information on the dynamics of stock returns processes in a very costless algorithmically way. An initial analysis of this information suggests the presence of strong nonlinear and possibly deterministic dynamics.

2   A different view of the compass rose

Price clustering and discreteness, as manifested in the compass rose, appears in the form of rays that emanate from the center of the phase portraits (see Fig. 1(a) were we have a detail of the compass rose for the returns of the TESCO stock, chosen randomly from stocks forming to the FTSE100 index). It has been suggested that there is no predictability in such phase portraits

(Crack and Ledoit, 1996). This is due to temporal structure and information being concealed in the pase portraits of return sequences. One could initially adopt the view that the distribution of the points in the compass rose in Fig. 1 could have been generated by a suitably discretized random Gaussian process. However (Antoniou and Vorlow, 2004b) showed that there may be a more delicate temporal pattern hidden under noise in the compass rose.

[Insert Fig. 1 about here.]

In this letter we follow the approach of Koppl and Nardone (2001) and instead of examining directly the distribution of points in the phase portraits, we choose to model the value of the arcs Φ formed by the line joining the point with the origin (0,0) and the horizontal axis (Fig. 1a). We measure this in degrees for simplicity. An advantage of such "recoding" of the compass rose information is that all prices will range between 0 and 2π or 0 and 360 degrees. It will also allow us to observe more efficiently the clustering and obtain a non-subjective view of the compass rose pattern (refer to Crack and Ledoit, 1996 and Koppl and Nardone, 2001 for discussions). Indeed, the investigation of the distribution of the compass rose arc values for TESCO (Fig. 1b), shows that there is concentration of points across the horizontal and vertical axis (0, 90, 180 and 270 (-90) degrees) and the diagonal rays (close to 45, 45+90, 45+180 and -45 degrees i.e., the NE-SW and NW-SE directions of the compass rose). This is referred to as *X-skewness* in Koppl and Nardone (2001) and attributed to the presence of big-players and herding (see Koppl and Yeager, 1996 and Broussard and Koppl, 1996). They also suggested that X-skewing may be inconsistent with ARCH effects and demonstrate this with simulations.[1] Mandelbrot (1999b,a) also revealed a similar result through a different approach. Judging from the above literature, it seems that several differently fabricated nonlinear and non-random sequences can pass as (G)ARCH processes. There is also the case that stock returns may be characterized by more complex dynamics, not excluding deterministic or chaotic structures (refer to Kyrtsou and Terraza, 2002, 2003 and Antoniou and Vorlow, 2000, 2004b,a for more details).

If we plot the arc values Φ as a time series we also obtain a pattern that confirms this clustering of points (see Fig. 1c). Plotting a sorted version of the sequence in Fig. 1(c), provides us with the pattern observed in 1(d), where the plateaus indicate more clearly where the clustering occurs. The intensity of the compass rose patterns may differ between stocks, however the clustering along the main directions as discussed above, usually prevails. Searching for

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[1] The multimodality of the distribution in the histogram of Fig. 1(b) may also be an indication of more complex dynamics. See in (Kantz and Schreiber, 1997) and (Kaplan and Glass, 1995) for discussions.

some type of temporal dependency, we looked into the phase portraits of the arcs. In Fig. 2(a) we have the first lag phase portrait and in Fig. 2(b) the second lag one. We can see very curious patterns arising in both diagrams. There are grids (Fig. 2b) and rays (Fig. 2a) which correspond to the main rays of the compass rose as seen in Fig. 1(c) and (d). These patterns need more analysis.

[Insert Fig. 2 about here.]

In order to provide evidence of some kind of dependency in the above patterns we calculated the BDS test (see Brock et al., 1987 and Brock and Baek, 1991) for the TESCO sequence of arc values Φ. In table 1 we see clearly that for a range of dimensions d_E (2 to 10), the BDS test reports non IID dynamics for neighborhood area ranges between 0.5 and 1.5 times the standard deviation of arcs. When the size becomes twice the standard deviation (which is regarded as a large radius), we can accept independence only up to dimension 3. This is an initial indication that the dynamics as observed from the sequence of arcs, may contain some dependency that could be used for forecasting purposes.

[Insert table 1 about here.]

To back up this result, we calculated the BDS test within a Surrogate Data Analysis framework (SDA: Theiler et al., 1992). SDA is a permutation test framework, similar to bootstrapping, and is used to test specific nulls that exclude certain dynamics. Every null comes with its own tailor made simulation procedures for creating surrogate data sets from the original sequences (refer to Kaplan and Glass, 1995, Kantz and Schreiber, 1997, Schreiber and Schmitz, 1996, 2000 and Kugiumtzis, 2001 for more details). In nutshell, via SDA one searches for large discrepancies between the statistic values on the original and surrogate sequences. When this occurs, one can safely reject the hypothesis that the original sequence stems from a process that is in compliance with the null. SDA is appropriate here as it allows us to bypass the limitations of chaotic invariance measures (such as Lyapunov exponents and dimension statistics) due to small data sets and noisy information. Although the BDS is a test for independence and not nonlinear determinism and complexity, under the SDA framework it can be used to test for absence of stochastic randomness via the exclusion of the null hypothesis.

For our case we used the null of the Φ sequences being *a monotonic nonlinear transformation of linearly filtered noise*, which is also regarded as the "most interesting". This implies strongly, the absence of stochastic (random) dynamics. We analyzed data from 53 FTSE 100 index stocks, spanning the period 01/01/1970 to 5/30/2003 (a maximum of 8717 observations). In tables 2 and

3 we present the results for 5% and 2.5% levels of statistical significance, for neighborhood sizes ϵ ranging from 0.5 to 2 times the standard deviation of Φ. We clearly see that for up to 1.5 times the standard deviation, the null can be safely rejected (observe the large biases). However, for ϵ equalling twice the standard deviation, we can not always reject the null (but this is only for a few cases). Our SDA results do not reject the case of forecastability on the basis of arc values. They also suggest the presence of complex dynamics and the possibility of some level of determinism.

[Insert table 2 about here.]

[Insert table 3 about here.]

3 Conclusions

By adopting an approach similar to Koppl and Nardone (2001) we showed that the compass rose (Crack and Ledoit, 1996) can provide useful information for understanding further the dynamics of stock return sequences. Moreover, we do not reject the case of these dynamics being complex and forecastable. We also do not exclude the possibility of nonlinear determinism. However more research is needed on this controversial area (see Andrew and MacKinlay, 1988, Mayfield and Mizrach, 1992, and Hsieh, 1991). An interesting area for future research would be to experiment with differently simulated processes (such as purely chaotic, stochastic, (G)ARCH and mixtures of these) and examine similarities or dissimilarities with the results for the framework we follow in this letter. It would also be interesting to see how inhomogeneous sampling of such processes could alter the results we report here. That could provide useful information on how our view of the stock return dynamics changes as we move from high-frequency data to lower frequencies, which is also an issue that has been concerning the general compass rose literature.

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Table 1

BDS test statistic results and p-values for the TESCO arc values Φ. Embedding dimensions d_E 2 to 10. Neighborhood size ϵ in terms of standard deviation s of Φ.

|	BDS Statistic			
ϵ:	$0.5 \times s$	$1.0 \times s$	$1.5 \times s$	$2.0 \times s$
d_E	0.897	1.794	2.691	3.588
2	745.31	241.38	65.39	3.45
3	846.67	223.27	56.12	2.84
4	1010.50	220.27	49.64	1.68
5	1243.98	222.22	44.97	0.95
6	1577.48	229.07	41.54	0.39
7	2053.94	239.21	38.59	−0.18
8	2743.93	252.22	36.10	−0.72
9	3741.75	269.69	34.16	−1.08
10	5201.71	291.13	32.85	−1.16
	BDS Statistic p-values			
2	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.09
5	0.00	0.00	0.00	0.34
6	0.00	0.00	0.00	0.69
7	0.00	0.00	0.00	0.86
8	0.00	0.00	0.00	0.47
9	0.00	0.00	0.00	0.28
10	0.00	0.00	0.00	0.25

Table 2
Surrogate Data Analysis results on arc values Φ for 53 companies in the FTSE100.
Discriminating statistic: BDS test (embedding dimension 2). Neighborhood size
$\epsilon_1 = 0.5 \times s$, $\epsilon_2 = 1.0 \times s$, $\epsilon_3 = 1.5 \times s$ and $\epsilon_4 = 2.0 \times s$, where $s =$ standard
deviation of Φ sequence. Biases and standard errors reported for significance level
$\alpha = 5\%$.

| Neighborhood size ϵ:	Statistic (BDS)				Bias				Standard Error			
	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4
FTSE ALL SHARE - PRICE INDEX	1249.61	339.45	48.02	−21.75	−1246.52	−338.09	−47.34	22.31	2.53	1.54	1.19	1.16
FTSE 100 - PRICE INDEX	152.69	97.72	35.55	20.52	−152.91	−97.70	−35.56	−20.58	0.87	1.00	0.94	1.21
ALLIED DOMECQ	823.28	259.24	68.38	1.29	−821.57	−258.47	−67.80	−0.93	1.81	1.00	0.99	1.06
AMVESCAP	159.36	130.18	60.63	28.46	−158.57	−129.63	−59.80	−27.64	1.20	1.24	1.26	1.25
ASSD.BRIT.FOODS	290.34	163.95	62.85	21.22	−289.08	−162.77	−61.89	−20.08	1.37	1.13	1.32	1.26
AVIVA	928.47	270.09	66.89	0.16	−927.36	−269.49	−66.61	0.11	1.38	0.96	1.17	1.13
BARCLAYS	1076.24	281.21	62.10	−5.50	−1076.56	−281.26	−62.33	5.20	1.21	1.16	1.09	1.13
BOC GROUP	829.83	257.29	67.76	3.58	−828.10	−256.45	−67.26	−3.16	1.70	1.05	0.91	0.92
BOOTS GROUP	1076.14	283.31	64.05	−5.67	−1076.77	−283.69	−64.38	5.31	1.01	0.92	0.79	0.81
BP	1115.66	295.08	63.04	−7.99	−1114.58	−294.74	−62.90	8.04	1.32	1.04	1.24	1.26
BRIT.AMERICAN TOBACCO	961.19	273.01	66.07	−3.63	−959.84	−272.50	−65.74	3.80	1.23	0.82	0.75	0.76
BRITISH LAND	568.83	223.68	66.58	4.10	−567.36	−223.00	−66.02	−3.65	1.02	0.94	0.72	0.74
BUNZL	192.12	138.33	59.76	23.61	−191.62	−137.83	−59.07	−23.05	1.01	0.97	1.18	1.10
CADBURY SCHWEPPES	653.11	233.87	70.17	6.34	−651.55	−232.74	−69.52	−5.83	1.25	0.79	0.70	0.68
DAILY MAIL 'A'	89.23	96.50	47.69	35.62	−88.25	−95.23	−46.43	−33.71	0.78	0.95	0.85	0.77
DIAGEO	619.63	225.74	67.63	5.33	−618.52	−224.98	−67.00	−4.83	1.01	0.88	0.84	0.71
DIXONS GP.	341.08	172.89	60.51	13.26	−340.48	−172.19	−60.22	−12.95	1.19	1.12	1.11	1.22
EMAP	72.70	69.19	35.03	20.02	−72.73	−68.86	−35.12	−20.23	1.26	1.27	1.26	1.07
EXEL	433.53	203.06	69.26	9.61	−433.38	−203.00	−69.29	−9.73	1.01	0.82	0.90	0.87
FOREIGN & COLONIAL	711.36	237.89	63.39	6.31	−709.53	−236.99	−62.93	−5.96	1.00	1.04	1.19	1.07
GKN	979.42	279.06	65.76	−2.93	−978.52	−278.57	−65.37	3.30	1.39	0.88	0.81	0.80
GLAXOSMITHKLINE	1069.02	282.23	62.10	−6.11	−1068.62	−282.08	−61.98	6.14	1.79	1.12	1.04	1.02
GRANADA	459.67	200.60	65.15	9.49	−459.69	−200.59	−65.08	−9.41	1.27	1.19	1.11	1.16
GUS	128.72	111.26	50.83	35.48	−128.49	−110.76	−50.36	−34.92	0.89	0.92	0.93	0.83
HANSON	804.72	251.67	61.16	−2.19	−804.64	−251.61	−61.11	2.20	1.06	1.09	0.90	0.98
HILTON GROUP	777.65	248.02	70.54	4.30	−775.18	−246.83	−69.75	−3.75	1.92	1.20	1.21	1.18
IMP.CHM.INDS.	1007.77	283.71	65.69	−5.80	−1007.17	−283.64	−65.66	5.77	1.64	0.92	1.22	1.21
JOHNSON MATTHEY	378.71	178.51	63.97	14.39	−378.47	−178.21	−63.81	−14.15	0.91	0.99	0.89	0.89
LAND SECURITIES	1089.61	282.28	62.79	−6.22	−1088.38	−281.71	−62.41	6.55	1.83	1.30	1.22	1.20
LEGAL & GENERAL	970.82	269.13	65.65	0.04	−969.14	−268.62	−65.34	0.22	1.09	1.10	0.88	0.92
MARKS & SPENCER GROUP	912.70	272.35	64.73	−2.03	−909.93	−270.77	−63.62	3.04	1.67	1.36	1.32	1.26
MORRISON (WM) SPMKTS.	90.75	90.64	45.66	44.19	−89.83	−89.01	−44.06	−41.71	0.93	0.97	0.76	1.07
NEXT	253.11	151.42	55.97	18.84	−252.81	−150.97	−55.62	−18.50	1.02	1.16	0.94	1.13
PEARSON	444.27	196.80	64.87	8.11	−444.02	−196.49	−64.80	−8.04	1.08	0.84	0.94	0.84
PROVIDENT FINL.	266.69	155.41	58.69	16.83	−266.57	−155.36	−58.50	−16.76	0.77	0.98	1.11	0.77
PRUDENTIAL	910.07	267.31	65.78	0.13	−909.09	−266.63	−65.29	0.37	1.08	0.98	0.79	0.79
RECKITT BENCKISER	836.20	258.41	67.44	2.10	−836.20	−258.47	−67.68	−2.30	1.33	1.29	1.19	1.24
REED ELSEVIER	898.32	261.80	63.61	−1.99	−898.15	−261.72	−63.60	1.93	1.37	1.14	1.04	1.01
RENTOKIL INITIAL	197.81	131.19	56.03	28.14	−196.12	−129.70	−54.49	−26.38	0.97	0.94	0.96	0.82
REXAM	697.01	237.79	72.49	6.51	−695.08	−236.83	−71.66	−5.71	1.38	0.90	0.90	0.88
RIO TINTO	849.08	260.87	66.48	−3.08	−848.51	−260.71	−66.40	3.19	1.59	1.37	1.33	1.31
ROYAL BANK OF SCOTLAND	433.58	198.72	65.90	9.33	−432.41	−198.07	−65.28	−8.83	0.88	1.06	0.93	0.86
SAINSBURY (J)	698.76	227.30	59.38	0.21	−698.30	−227.22	−59.18	−0.21	1.15	1.29	1.29	1.21
SCHRODERS	72.28	84.11	40.53	41.80	−71.93	−83.50	−39.86	−40.50	1.14	1.17	1.16	1.20
SCOT. & NEWCASTLE	750.36	245.66	67.65	0.87	−749.30	−244.78	−67.08	−0.40	1.09	0.72	0.94	0.95
SHELL TRANSPORT & TRDG.	1199.79	294.59	65.20	−6.32	−1198.76	−293.78	−64.65	6.72	1.46	1.07	1.04	1.04
SMITH & NEPHEW	500.14	208.90	71.11	14.25	−497.28	−206.89	−69.41	−12.80	1.64	1.67	1.60	1.60
SMITHS GROUP	587.34	222.17	69.99	10.48	−586.92	−222.11	−70.07	−10.66	1.36	0.92	1.27	1.29
STD.CHARTERED	623.16	223.03	67.43	5.56	−622.41	−222.38	−66.93	−5.16	1.24	1.05	0.84	0.93
TESCO	745.31	241.38	65.24	3.24	−744.56	−240.87	−64.96	−2.97	1.55	1.15	1.15	1.14
TOMKINS	77.66	73.89	37.40	25.49	−77.65	−73.70	−37.12	−25.38	1.17	1.29	0.94	0.93
UNILEVER (UK)	1017.26	278.21	59.93	−7.77	−1017.15	−278.20	−59.90	7.76	1.39	1.04	1.09	0.98
WHITBREAD	851.02	262.03	66.73	3.23	−849.13	−260.92	−65.95	−2.46	1.79	1.20	1.11	1.11
WOLSELEY	172.73	129.30	52.87	27.14	−172.66	−129.15	−52.69	−27.06	0.82	0.95	1.03	0.89
WPP GROUP	71.43	60.94	38.30	21.64	−71.01	−60.48	−37.74	−21.25	1.11	0.81	0.82	0.82

Table 3
Surrogate Data Analysis results on arc values Φ for 53 companies in the FTSE100.
Discriminating statistic: BDS test (embedding dimension 2). Neighborhood size
$\epsilon_1 = 0.5 \times s$, $\epsilon_2 = 1.0 \times s$, $\epsilon_3 = 1.5 \times s$ and $\epsilon_4 = 2.0 \times s$, where $s =$ standard
deviation of Φ sequence. Biases and standard errors reported for significance level
$\alpha = 2.5\%$.

| | Statistic (BDS) | | | | Bias | | | | Standard Error | | | |
Neighborhood size ϵ:	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_1	ϵ_2	ϵ_3	ϵ_4
FTSE ALL SHARE - PRICE INDEX	1249.61	339.45	48.02	−21.75	−1245.98	−338.20	−47.50	22.15	1.97	0.98	0.98	1.01
FTSE 100 - PRICE INDEX	152.69	97.72	35.55	20.52	−152.69	−97.82	−35.55	−20.54	0.98	0.93	0.86	1.12
ALLIED DOMECQ	823.28	259.24	68.38	1.29	−821.92	−258.54	−68.02	−1.07	1.35	0.96	0.97	0.92
AMVESCAP	159.36	130.18	60.63	28.46	−158.41	−129.22	−59.67	−27.18	0.90	0.83	0.85	0.89
ASSD.BRIT.FOODS	290.34	163.95	62.85	21.22	−288.64	−162.40	−61.51	−19.84	1.35	1.33	1.37	1.30
AVIVA	928.47	270.09	66.89	0.16	−926.88	−269.44	−66.56	0.05	1.36	1.18	1.23	1.17
BARCLAYS	1076.24	281.21	62.10	−5.50	−1075.93	−281.31	−62.19	5.38	1.36	1.14	1.16	1.11
BOC GROUP	829.83	257.29	67.76	3.58	−828.19	−256.50	−67.12	−3.08	1.62	1.03	0.89	0.84
BOOTS GROUP	1076.14	283.31	64.05	−5.67	−1075.91	−283.18	−63.98	5.69	1.26	1.01	0.95	0.92
BP	1115.66	295.08	63.04	−7.99	−1114.62	−294.54	−62.78	8.18	1.70	1.34	1.19	1.20
BRIT.AMERICAN TOBACCO	961.19	273.01	66.07	−3.63	−959.60	−272.40	−65.67	3.99	1.37	1.08	1.00	0.94
BRITISH LAND	568.83	223.68	66.58	4.10	−567.30	−222.67	−65.59	−3.19	1.17	0.95	1.03	1.00
BUNZL	192.12	138.33	59.76	23.61	−191.61	−137.79	−59.05	−22.97	0.99	0.98	0.90	0.84
CADBURY SCHWEPPES	653.11	233.87	70.17	6.34	−651.52	−232.88	−69.47	−5.61	1.35	1.04	1.02	1.00
DAILY MAIL 'A'	89.23	96.50	47.69	35.62	−88.45	−95.38	−46.60	−33.87	1.07	0.99	1.01	1.14
DIAGEO	619.63	225.74	67.63	5.33	−618.36	−225.24	−67.24	−5.02	1.26	1.15	1.19	1.22
DIXONS GP.	341.08	172.89	60.51	13.26	−340.07	−172.18	−59.87	−12.58	0.97	0.99	0.88	0.99
EMAP	72.70	69.19	35.03	20.02	−72.78	−68.80	−34.77	−20.02	1.04	1.08	0.87	1.04
EXEL	433.53	203.06	69.26	9.61	−433.28	−202.75	−69.08	−9.38	1.04	1.04	0.99	0.97
FOREIGN & COLONIAL	711.36	237.89	63.39	6.31	−709.87	−236.98	−62.74	−5.78	1.23	1.21	1.03	1.08
GKN	979.42	279.06	65.76	−2.93	−978.64	−278.57	−65.46	3.20	1.33	1.10	1.04	1.08
GLAXOSMITHKLINE	1069.02	282.23	62.10	−6.11	−1068.51	−281.94	−61.86	6.30	1.29	1.03	1.03	1.04
GRANADA	459.67	200.60	65.15	9.49	−459.69	−200.39	−65.16	−9.38	1.15	1.29	1.14	1.22
GUS	128.72	111.26	50.83	35.48	−128.32	−110.95	−50.35	−34.85	1.15	1.12	1.14	1.09
HANSON	804.72	251.67	61.16	−2.19	−804.75	−251.79	−61.30	2.07	1.09	1.03	0.99	1.02
HILTON GROUP	777.65	248.02	70.54	4.30	−776.09	−247.12	−69.98	−3.90	1.31	1.17	1.10	1.14
IMP.CHM.INDS.	1007.77	283.71	65.69	−5.80	−1007.48	−283.85	−65.79	5.67	1.33	1.03	0.88	0.88
JOHNSON MATTHEY	378.71	178.51	63.97	14.39	−378.02	−177.88	−63.54	−13.91	1.03	1.01	0.89	0.88
LAND SECURITIES	1089.61	282.28	62.79	−6.22	−1088.71	−281.87	−62.38	6.55	1.26	1.10	0.99	0.95
LEGAL & GENERAL	970.82	269.13	65.65	0.04	−969.44	−268.38	−65.24	0.33	1.36	1.08	0.96	0.96
MARKS & SPENCER GROUP	912.70	272.35	64.73	−2.03	−909.91	−270.94	−63.82	2.77	1.33	0.95	0.78	0.85
MORRISON (WM) SPMKTS.	90.75	90.64	45.66	44.19	−90.24	−89.74	−44.68	−42.54	0.93	0.90	0.92	0.93
NEXT	253.11	151.42	55.97	18.84	−252.66	−151.03	−55.45	−18.50	0.82	0.96	0.82	0.73
PEARSON	444.27	196.80	64.87	8.11	−443.83	−196.62	−64.64	−7.87	1.09	1.02	1.00	0.94
PROVIDENT FINL.	266.69	155.41	58.69	16.83	−266.55	−155.49	−58.70	−16.89	1.04	1.29	1.20	1.09
PRUDENTIAL	910.07	267.31	65.78	0.13	−909.18	−267.05	−65.59	−0.00	1.22	1.01	1.06	1.11
RECKITT BENCKISER	836.20	258.41	67.44	2.10	−835.56	−257.93	−67.04	−1.72	1.20	1.12	1.00	0.98
REED ELSEVIER	898.32	261.80	63.61	−1.99	−897.92	−261.77	−63.47	2.14	1.37	1.13	1.12	1.13
RENTOKIL INITIAL	197.81	131.19	56.03	28.14	−196.05	−129.48	−54.33	−26.04	0.91	1.19	1.09	1.17
REXAM	697.01	237.79	72.49	6.51	−695.26	−236.76	−71.64	−5.70	1.53	1.21	1.17	1.21
RIO TINTO	849.08	260.87	66.48	−3.08	−848.18	−260.53	−66.23	3.30	1.03	0.98	0.85	0.86
ROYAL BANK OF SCOTLAND	433.58	198.72	65.90	9.33	−432.32	−198.01	−65.34	−8.82	1.15	0.96	0.81	0.83
SAINSBURY (J)	698.76	227.30	59.38	0.21	−698.48	−227.33	−59.46	−0.22	0.93	0.95	0.83	0.82
SCHRODERS	72.28	84.11	40.53	41.80	−71.77	−83.11	−39.59	−40.11	1.03	1.13	1.10	1.26
SCOT. & NEWCASTLE	750.36	245.66	67.65	0.87	−749.08	−245.00	−67.06	−0.26	1.03	0.96	0.91	0.89
SHELL TRANSPORT & TRDG.	1199.79	294.59	65.20	−6.32	−1198.60	−294.14	−65.03	6.45	1.58	1.17	1.10	1.07
SMITH & NEPHEW	500.14	208.90	71.11	14.25	−497.73	−207.15	−69.72	−12.99	1.17	1.20	1.06	1.11
SMITHS GROUP	587.34	222.17	69.99	10.48	−586.43	−221.51	−69.40	−9.90	1.28	0.97	1.12	1.14
STD.CHARTERED	623.16	223.03	67.43	5.56	−623.18	−222.82	−67.38	−5.56	1.18	1.10	1.13	1.21
TESCO	745.31	241.38	65.24	3.24	−745.08	−241.37	−65.23	−3.33	1.12	1.16	1.10	1.08
TOMKINS	77.66	73.89	37.40	25.49	−77.72	−73.55	−37.02	−25.33	0.87	0.98	1.10	0.90
UNILEVER (UK)	1017.26	278.21	59.93	−7.77	−1017.42	−278.30	−60.03	7.68	1.28	0.98	0.98	0.96
WHITBREAD	851.02	262.03	66.73	3.23	−849.28	−261.35	−66.25	−2.81	1.44	1.10	1.12	1.16
WOLSELEY	172.73	129.30	52.87	27.14	−172.58	−129.19	−52.74	−26.87	1.06	1.08	1.10	0.96
WPP GROUP	71.43	60.94	38.30	21.64	−71.32	−60.89	−38.02	−21.43	1.00	1.01	0.73	0.94
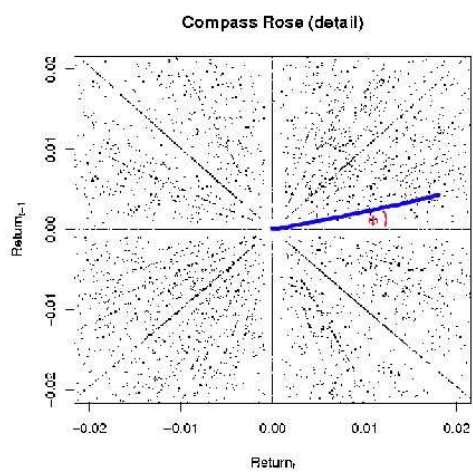
(a) Compass Rose.
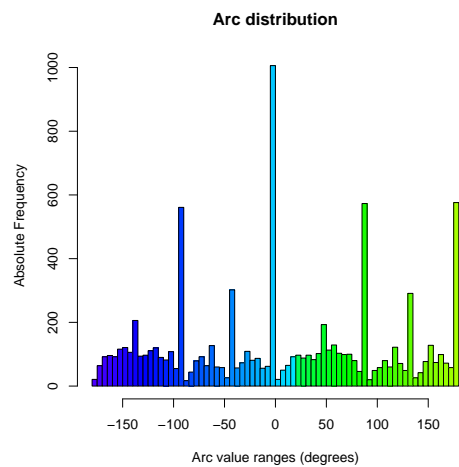
(b) Arc Distribution.
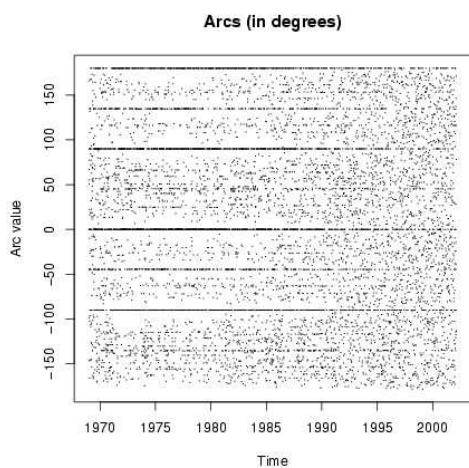
(c) Arc values (unsorted).
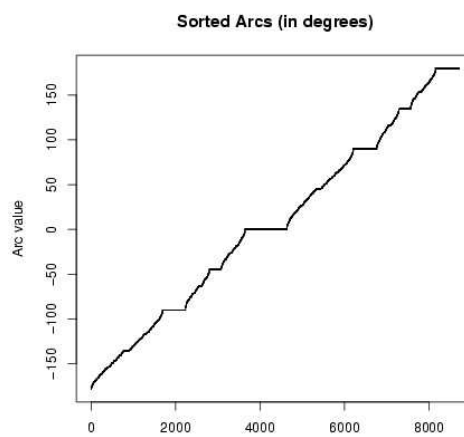
(d) Arc values (sorted).

Fig. 1. Patterns observed in the arc distribution.

TESCO compass rose arcs phase portrait

(a) Lag 1.

TESCO compass rose arcs phase portrait
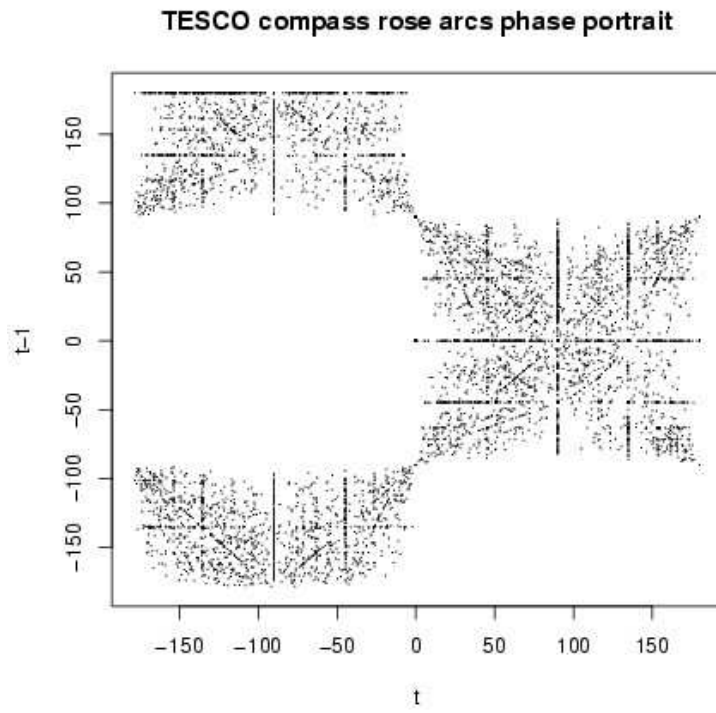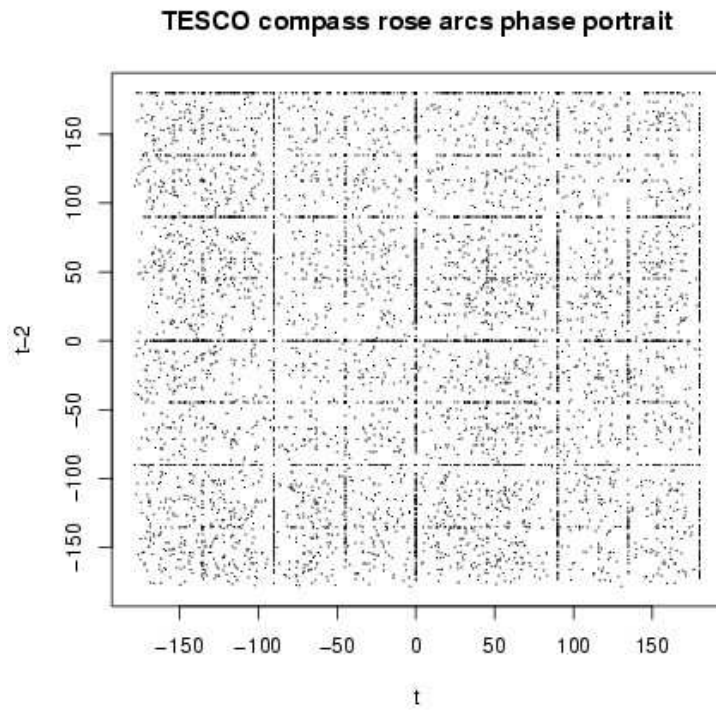
(b) Lag 2.

Fig. 2. Patterns observed in the arc phase portraits.